Superconductivity in the presence of spin-orbit and spin-splitting interactions: magneto-electric and spin galvanic effects

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This research originates in a try to understand spin-orbit effect in superconductors. Since a few years, there have been growing interests in this kind of interaction, mainly in order to generate the so-called topological effect in superconductors and the unpaired Majorana modes in superconducting wires (Franz, 2013; Wilczek, 2009). Nevertheless, this rush towards topological effect might have forgotten several interesting aspects of the association of spin-orbit and spin-splitting (i.e. Zeeman interaction) effects in superconductor phenomenology. This presentation is dedicated to such an attempt.

I will discuss the gauge-covariant transport formalism, a novel tool to put the spin-orbit effect into some semi-classic limit in a systematic way. I then apply this formalism to the study of a Josephson junction and discuss two distinct regimes in this system. The first one can be seen as existing in short junction only. I will show how the spin texture modifies the (quasi-classic) Green function in term of a spin holonomy due to the perfect Andreev reflections. For longer junctions, a spin-Hall like effect starts to be of importance. The spin-Hall and reverse spin-Hall (generation of a spin polarization induced by a charge current and the generation of a charge current due to the spin-polarization) then conspire to generate a static, equilibrium charge current in a Josephson junction. It results a magneto-electric φ_0 phase-shift proportional to the spin-orbit and spin-splitting effects.

The slides accompanying this talk can be found on my personnal webpage.

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I. GAUGE-COVARIANT TRANSPORT FORMALISM

The usual transport formalism consists in an expansion of the Green function from the quantum-field-theory to the low-energy sector of the phase-space, using a Wigner transform

$$G(p,x)=\int dz\left[e^{-\mathbf{i}p\cdot z/\hbar}G\left(x-\frac{z}{2},x+\frac{z}{2}\right)\right] \tag{1.1}$$

where the two point-correlator $G(x_1=x-z/2,x_2=x+z/2)$ on the right-hand-side is converted to a one-point correlator in the phase space $G(p,x)$ via a partial Fourier transformation of the relative coordinate, then keeping the center-of-mass coordinate and the momentum as variables. When truncated at order $\mathcal{O}(\hbar^2)$ for instance, the Fourier transform is called a semi-classic expansion. The equation for $G(p,x)$ at this order is called the transport equation, see (Rammer and Smith, 1986).

For system with spin-orbit interaction, though, the Hamiltonian reads (in the case of Rashba spin-orbit interaction and a quadratic band dispersion below)

$$\begin{aligned} H &= \frac{p_x^2}{2m}+\frac{p_y^2}{2m}-\mu+\boldsymbol{h}\cdot\boldsymbol{\sigma}+v_{\text{so}}(\boldsymbol{\sigma}\times\boldsymbol{p})\cdot\hat{\boldsymbol{z}} \\ &= \frac{(p_x-mv_{\text{so}}\sigma_y)^2}{2m}+\frac{(p_y+mv_{\text{so}}\sigma_x)^2}{2m}-\tilde{\mu}+A_0 \end{aligned} \tag{1.2}$$

with $\boldsymbol{h}\cdot\boldsymbol{\sigma}$ the exchange (Zeeman) interaction, $p_{x,y}$ the 2D momentum, and v_{so} the velocity associated to the spin-orbit interaction. The difficulty – as seen in the first line above – is the p-dependent component of the Hamiltonian, which is now intrinsically dressed by a spin texture, see $v_{\text{so}}(\boldsymbol{\sigma}\times\boldsymbol{p})\cdot\hat{\boldsymbol{z}}$. Nevertheless – see the second line – one can rewrite this Hamiltonian in the form of a gauge-covariant one, when the spin-orbit interaction is seen as a gauge-potential

$$\boldsymbol{A}=\xi_{\text{so}}^{-1}(-\sigma_y,\sigma_x,0)\ ;\ \xi_{\text{so}}=\hbar/mv_{\text{so}} \tag{1.3}$$

in this example. This rewriting just renormalizes the chemical potential $\tilde{\mu}=\mu+mv_{\text{so}}^2/2$ in the example of Rashba spin-orbit interaction for free electrons. Note that $\boldsymbol{A}$ is in fact a vector in the real space (noted with lower indices) $A_x=-\xi_{\text{so}}^{-1}\sigma_y$, $A_y=\xi_{\text{so}}^{-1}\sigma_x$ but also a vector in the spin-space (noted with upper index) $A_x^x=-A_y^x=\xi_{\text{so}}^{-1}$. Then the Hamiltonian reads $H=(\boldsymbol{p}+\boldsymbol{A})^2/2m-\tilde{\mu}+A_0$, where $A_0=\boldsymbol{h}\cdot\boldsymbol{\sigma}$ represents the spin-splitting effects (i.e. Zeeman interaction), seen as the time-sector of the gauge-potential (A_0 is a scalar in the real space, and a vector in the spin-space $A_0^x=h_x$, ...).

Now we can promote the simple rewriting above as a recipe for the spin interaction. In this situation, the spin is seen as a redundant degree of freedom, which can be rotated independently at different points of the space. It means in particular that the gauge-covariance (seen as a position dependent spin-rotation $R(x)$) for the Green function reads

$$G(x_1,x_2)\rightarrow R(x_1)\,G(x_1,x_2)\,R^{\dagger}(x_2) \tag{1.4}$$

, i.e. it can be done differently for the initial x_1 and final point x_2 of the correlator. When injected into the Wigner transform (1.1), the gauge-covariance is broken due to the Fourier transform of the relative coordinate z. Since we promote the gauge-covariance as a general principle of the dynamics of our system, one needs a gauge-covariant Wigner transform, which in fact reads

$$G(p,x)=\int dz\left[e^{-\mathbf{i}p\cdot z/\hbar}U\left(x,x-\frac{z}{2}\right)G\left(x-\frac{z}{2},x+\frac{z}{2}\right)\left(x+\frac{z}{2},x\right)\right] \tag{1.5}$$

with the parallel displacement operation

$$U(b,a) = \mathrm{Pexp}\left[-\mathbf{i}\int_a^b dz^\mu A_\mu(z)\right] \tag{1.6}$$

guaranteeing that $G(p,x) \rightarrow R(x)\,G(p,x)\,R^\dagger(x)$ transforms as a single point operation under a gauge transformation. The cost for maintaining the gauge-covariance is the introduction of the path-ordered displacement operator U from the relative coordinate $x_1 = x - z/2$ back to the center-of-mass one x before taking the Fourier averaging into account.

An other important aspect of the formalism is the gauge-invariance of the BCS Hamiltonian with respect to a gauge-transform (= a spin rotation). This is true only for the s-wave superconductivity, when the Cooper pairs have singlet symmetry in spin. SO the transport method applies only straighforwardly to an s-wave superconductor with linear-in-momentum spin-orbit interaction.

Equipped with the gauge-covariant Wigner transform, one can write some gauge-covariant transport equation using standard procedure (Konschelle, 2014). One obtains (the check $\check{\cdots}$ on top of the quantity represents the Nambu space, or electron-hole redundancy of the theory)

$$-\mathbf{i}\frac{p_i}{m}\tilde{\nabla}_i\check{G} + \left[\tau_3(\omega + A_0) + \check{\Delta}, \check{G}\right] + \frac{\mathbf{i}}{2}\left\{\tau_3 F_{0j} + v_i F_{ij}, \frac{\partial\check{G}}{\partial p_j}\right\} = \left[\frac{\mathbf{i}\,\langle\check{g}\rangle}{2\tau}, \check{G}\right] \tag{1.7}$$

with a covariant derivative

$$\tilde{\nabla}_i\check{G} = \frac{\partial\check{G}}{\partial x_i} + \mathbf{i}\left[\check{A}_i, \check{G}\right] \tag{1.8}$$

keeping track of the spin-orbit interaction and its gauge interpretation, τ the mean free time between two (spinless) impurity, $\langle\cdots\rangle$ some angular averaging in the momentum space (equivalently $\langle\check{g}\rangle$ is a kind of a Boltzmann integral/collision term), $\check{\Delta}$ is the gap parameter, obtained self-consistently from the semi-classic Green function $\check{G}(p,x)$, and

$$F_{\alpha\beta} = \frac{\partial A_\alpha}{\partial x_\beta} - \frac{\partial A_\beta}{\partial x_\alpha} - \mathbf{i}\left[A_\alpha, A_\beta\right] \tag{1.9}$$

is the gauge-field. The last term in $F_{\alpha\beta}$ disappears only in the Abelian case, when A_α are vectors in the real space only, and not vectors in both the real and spin spaces. F_{0i} is called the electric-like field, and F_{ij} is called the magnetic-like field.

An extra approximation can be used in metal when the Fermi energy is far larger than the superconducting gap: $E_F \gg \Delta$. It consists in supposing all the energies to be essentially fixed at the Fermi one, and to neglect the variation far away from E_F. Then one can define a $\varepsilon = v_F(p - p_F)$ integration of the energies (Ω_F is the angle in the momentum space, now fixed on the Fermi surface, since ε is only the radial direction)

$$\check{g}(p_F, \Omega_F, x) = \int \frac{\mathbf{i}d\varepsilon}{\pi}\check{G}(p,x) \tag{1.10}$$

for which the equation is

$$-\mathbf{i}v_F\left(n_i\tilde{\nabla}_i\right)\check{g} + \left[\tau_3(\omega + A_0) + \check{\Delta}, \check{g}\right] + \frac{\mathbf{i}}{2m}\left\{n_i F_{ij}, \frac{\partial\check{g}}{\partial n_j}\right\} = \left[\frac{\mathbf{i}\,\langle\check{g}\rangle}{2\tau}, \check{g}\right] \tag{1.11}$$

with $p_i/m = n_i v_F$. This last equation is a generalized Eilenberger equation or quasi-classic transport equation, which allows to describe spin-orbit (in the gauge-covariant derivative and spin-splitting (the time-sector A_0) interactions in the presence of superconducting correlation $\check{\Delta}$ and impurities $\langle \check{g} \rangle /2\tau = \int \hat{g} d\Omega_F/2\tau$. It is in principle self-consistent in $\check{\Delta}$ and impurities.

At this level of approximation, the electric-like field disappears, see details in (Konschelle *et al.*, 2015). We will see in section III that the magnetic-like field F_{ij} is responsible for the anomalous phase-shift in long enough junctions.

II. JOSEPHSON JUNCTION WITH MAGNETIC TEXTURE

We now discuss Josephson system using the Eilenberger equation (1.11). The Josephson junction is made of two superconducting regions connected via a normal one. The normal region has no superconducting gap $\check{\Delta} = 0$ there and magnetic interactions described by A_0 and $\boldsymbol{n} \cdot \boldsymbol{A}$ (in the covariant derivative). There is no magnetic interaction in the superconductors. When the normal region is ballistic, one has $\tau \to \infty$. In addition for a short junction (this hypothesis will be justified a posteriori, at the end of section (1.11)) one can suppose $F_{ij} \to 0$.

A Josephson junction is characterized by the phenomenology of the Andreev reflection, when an incident electron at an interface between a normal and a superconducting region is reflected back as a hole. It results a loop in a composite space made of the junction length and the Nambu space. Along the Nambu space, the space is discrete, since it consists only on positive and negative discrete charge which are interchanged via an Andreev reflection. Then starting at any point in the normal region, an electron will hit one interface where it will be reflected back as a hole, propagating in the other direction toward the other interface where it is transmuted as an electron again via a second Andreev reflection. It then comes back to its initial position after completing a so-called Andreev-Wilson loop.

When one wants to describe some spin effects, one can add an extra dimension to the Andreev-Wilson loop. When traveling across the junction, the complicated spin-structure might induce an amazingly complicated spin-dynamics. Nevertheless, the dynamics is constrained by the restriction of the loop: the particle must necessarily end-up in the same state (electron or hole) and at the same position as the initial ones. Then the initial and final spins can be compared. However complicated could the spin dynamics be, the comparison of the initial and final spin states can be described by two parameters, which are the angle Φ of the resulting rotation between the initial and final spin directions, and the vector $\mathfrak{n}$ around which the rotation is performed. So the Andreev-Wilson loop in the junction will give an extra contribution $W = e^{\mathbf{i}(\mathfrak{n}\cdot\boldsymbol{\sigma})\Phi}$ due to the spin effects.

To verify that the above simple analysis gives indeed the complete description of the spin-interaction, we calculated the Green function in the quasi-classic approximation. It reads

$$g = -\frac{\mathbf{i}}{2} \sum_{\alpha=\pm} (1 + \alpha (\mathfrak{n}(\boldsymbol{s}) \cdot \boldsymbol{\sigma})) \tan \left(\frac{\omega L}{v_F} + \arcsin \frac{\omega}{\Delta} + \frac{\varphi}{2} + \alpha \frac{\Phi}{2} \right) \tag{2.1}$$

giving the spectrum of Andreev-bound states (the poles of the Green function)

$$\frac{\omega L}{v_F} + \arccos \frac{\omega}{\Delta} \pm \frac{\varphi}{2} \pm \frac{\Phi}{2} = n\pi \tag{2.2}$$

where L is the length of the junction. There are four branches $\pm\varphi$ and $\pm\Phi$, with Φ the angle associated to the complicated spin-dynamics induced by the propagators

$$u(s_2, s_1) = \text{Pexp}\left\{-\mathbf{i}\int_{s_1}^{s_2} \left(A_0 + v_F\left(\boldsymbol{n}\cdot\boldsymbol{A}\right)\right) ds\right\} \tag{2.3}$$

writen in terms of a path-ordered integral. The complete loop reads

$$W(s) = u(s, s_R)\, \bar{u}(s_R, s_L)\, u(s_L, s) = e^{\mathbf{i}(\mathfrak{n}\cdot\boldsymbol{\sigma})\Phi} \tag{2.4}$$

with an electronic propagation from position s to the left interface at position s_L, then a hole-propagation given by the time-reversal $\bar{u}$ from the left to the right interface at position s_R and finally back to the initial point following an electronic propagation $u(s, s_R)$ from the right interface. Note that the angle appearing in the spectrum is defined as

$$2\cos\Phi = \text{Tr}\left\{e^{\mathbf{i}(\mathfrak{n}\cdot\boldsymbol{\sigma})\Phi}\right\} = \text{Tr}\left\{u(s_L, s_R)\, \bar{u}(s_R, s_L)\right\} \tag{2.5}$$

and is thus position independent. In contrary, the vector $\mathfrak{n}$ depends on the position, and can be seen as the axis around which the spin precesses along the junction.

In a pure mono-domain superconducting-ferromagnetic Josephson junction, one simply has $\Phi \propto 2hL/v_F$, the phase-shift associated to the exchange field h.

Since the u's are path-ordered exponentials, one can not calculate them in general. Nevertheless, one realizes that the Josephson junction is in fact calculating this complicated expression by itself. Then, if one is able to records the density-of-state, one can have an access to the angle Φ. The vector $\mathfrak{n}$ gives the full spin-structure of the Green-function. If one is able to measure the spin-polarized density of state, one will have a measure of this vector, and thus of the full mathematical object W. Then a Josephson junction is nothing but an analog computer of Wilson loop and path-ordered Dyson's series.

One can generalize a bit the spectrum, supposing a scattering region inside the junction, with transmission coefficient $T = 1 - R$. One obtains then

$$\cos\left(\frac{2\omega L}{v_F} + \alpha\Phi + 2\arcsin\frac{\omega}{\Delta}\right) + T\cos\varphi + R\cos\left(\frac{2\omega L}{v_F} + \alpha\Phi\right) = 0 \tag{2.6}$$

as the condition for the spectrum. We focus again in the short junction limit, when $\omega L/v_F \to 0$ (but keeping the Φ-term, even if it's a bit inconsistent since $\Phi \propto L$ as well) When $T \to 0$, one obtains the pure ballistic limit, when the energies are degenerate at high-symmetry points, for instance when $\varphi = \pi \pm \Phi$. These degeneracies are usually split by lowering the transparency T. Nevertheless, for a peculiar value $\Phi = \pi/2$, the degeneracies do not disappear, and one is left with quasi-zero energy Andreev-bound states at low transparency. The other degeneracy between $\downarrow$ and $\uparrow$ spin projection is always lifted by lowering the transparency. One of the spin-projection always goes to the bulk energy $\omega \to \pm\Delta$ at low transparency of the junction.

III. ANOMALOUS PHASE SHIFT AND MAGNETO-ELECTRIC RESPONSE

All section II was dedicated to the short junction limit, when the gauge-field in the Eilenberger equation can be safely neglected. We now explore the phenomenology associated to the gauge-field. In fact, from the Abelian limit (the usual Maxwell electromagnetism) one can infer many of this phenomenology.

For instance, we retained only the magnetic-like effect. In the Abelian Maxwell formalism, the magnetic field essentially tilts the electrons trajectories, which results in the Hall physics, when a current generates a charge accumulation in the transverse direction when a magnetic field is applied. Here in the non-Abelian generalization, one still expects some kind of tilted trajectories. This is the reason why a short enough junction will not exhibit the phenomenology of the gauge-fields, since the trajectories are correctly approximated by straight lines in this limit (one can talk of trajectories only in the ballistic limit, in the diffusive limit the argument is a bit different, but still the gauge-field is of higher order correction in the parameter τ, the mean free time, than the spin-splitting A_0 and spin-orbit $\boldsymbol{A}$ effects).

We also expect some kind of Hall-like phenomenology associated to the gauge-field. Since we have the non-Abelian version related to the spin effects of the gauge-theory, one expects that a spin-texture may generate spin-current, that spin-current induce spin-polarization, and that spin and charge effects will combine in some magneto-electric response.

For instance, if one starts with a bare charge and we let it propagate along the magnetic junction, it will first be dressed with a spin (i.e. it will acquire a spin polarization). At a little bit longer junction, the spin-orbit effect will start dressing the particle with a spin-texture, intrinsically momentum dependent. Next the gauge-field enters the stage, and collapses all these effects in order to generate a momentum (recall the Lorentz-like force appears with a momentum derivative in the transport equation). This momentum generation by a spin-orbit coupling and spin-dressing is called an inverse Edelstein effect, when a charge current is generated by a spin-polarization. In a Josephson junction, it is responsible for the generation of a spontaneous current at zero-phase difference. Equivalently it generates a current-phase relation with an anomalous phase-shift φ_0:

$$j = j_c \sin\left(\varphi - \varphi_0\right) \tag{3.1}$$

where the phase-shift is proportional to the spin-current J_x^a and the spin-polarization A_0^a:

$$j_x\left(\varphi = 0\right) \propto -\frac{e\pi N_0 v_F}{E_F}\frac{L^3}{3}\Delta^2 T_c F_{xi}^a F_{i0}^a \tag{3.2}$$

with $\mathrm{Tr}\left\{F_{xk}F_{k0}\right\} = J_x^a A_0^a$.

Usually this phase shift is difficult to measure, since in general the phase-difference adapts such that $\varphi = \varphi_0$ and no current should flow for single junction geometry. But one can design interference experiments when the phase-shift can be tuned and measure.

Many more details can be found in (Konschelle *et al.*, 2015). In particular, we here focused on the ballistic limit, but the φ_0 effect is in fact inherent to the presence of spin-orbit and spin-splitting effects. We calculated the magneto-electric anomalous phase shift for both the ballistic and diffusive limits, and for different interface transparency to prove it is a robust effect of generic appearance.

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